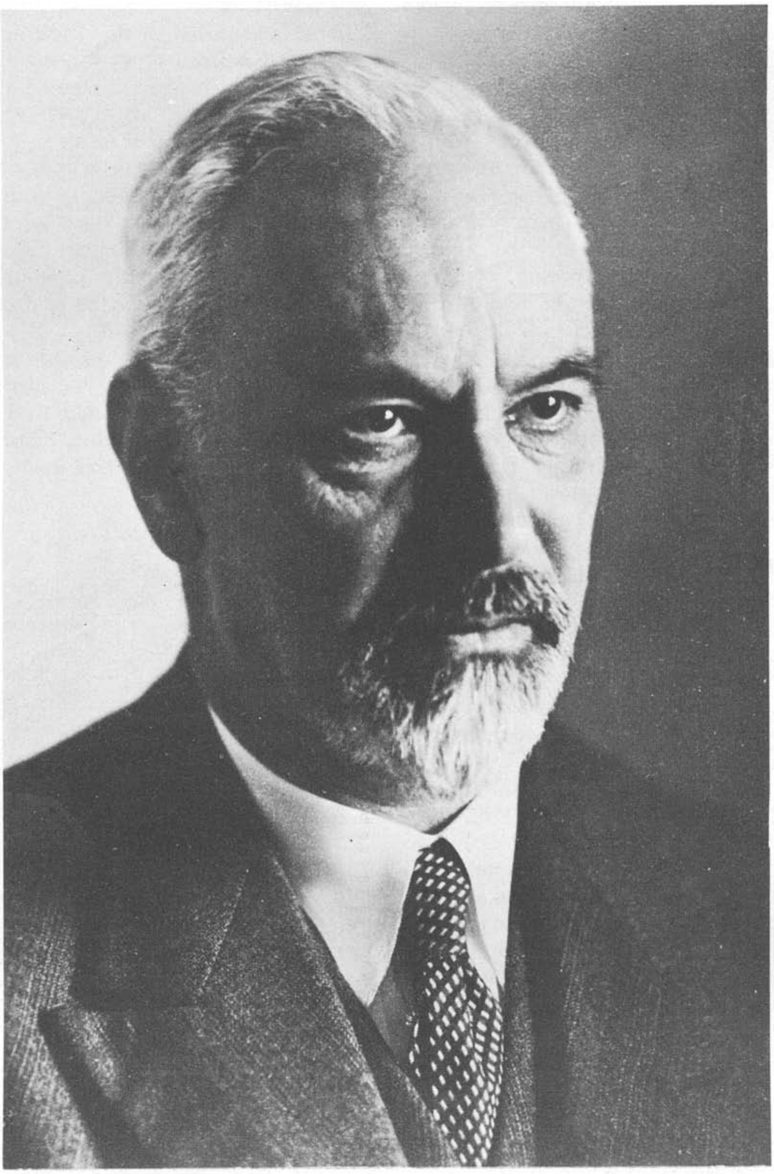
Int. J. Heat Mass Transfer. Vol. 18, pp. 1333–1336. Pergamon Press 1975. Printed in Great Britain



Ludwig Prandtl, 1875–1953

TRIBUTE TO LUDWIG PRANDTL

On 4 February of this year was the 100th anniversary of the birth of Ludwig Prandtl. The theory of convective heat transfer could not have been developed without the foundation of modern fluid mechanics created by him. We asked therefore Professor Hermann Schlichting, his student and close associate, to review briefly the work of Ludwig Prandtl.

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1. INTRODUCTION

Ludwig Prandtl (1875–1953) is generally accepted to be the father of modern fluid dynamics.

Since about the beginning of this century modern research in the field of fluid dynamics has achieved great success. It has been able to provide a theoretical clarification of observed phenomena which the science of classical hydrodynamics of the preceding century failed to do. Ludwig Prandtl has a major part in this achievement. There are essentially three branches of fluid dynamics which have become particularly well developed during the last seventy years and Prandtl has created the basis of all of them; they include boundary layer theory, aerofoil theory and gas dynamics.

On the occasion of the hundredth anniversary of Prandtl's birth a very condensed survey on his scientific life is given here.

Above is a photograph of Prandtl when he was about sixty years old. Prandtl was born on 4 February 1875 in the town of Freising in Bavaria; he died on 15 August 1953 in Göttingen at the age of nearly 79 years. In the year 1900, when Prandtl was 25 years old, he was awarded his doctorate at the University of Munich under August Foeppl. In the year 1901, after he had worked in industry at M.A.N. in Nürnberg for one year, that is at the age of 26 years, he was invited to join the Faculty of the Technical University at Hanover as Professor of mechanics.

At M.A.N. he was given the task of improving the design of an air duct for the removal of chips. In a manner typical for his approach to problems, he first measured the energy losses which occurred in the individual parts of the duct. This allowed him to recognize that the largest losses were connected with a conical portion which failed to produce the pressure recovery expected on the basis of Bernoulli's equation. This problem led him to the phenomenon of flow separation, which could be explained by his boundary layer theory. The corresponding paper, read in 1904 at the International Congress of Mathematics, made Prandtl famous at one stroke. The young Prandtl's achievements were soon recognized by Felix Klein, the great organizer of the mathematical and applied sciences at the University of Göttingen. In 1904 at Klein's initiative, Ludwig Prandtl took over the newly founded Institute of Applied Mechanics at Göttingen University. Based on this Institute and on the chair of the same name connected with it, Prandtl developed a most fruitful activity in research and teaching.

In the following I shall restrict myself exclusively to a consideration of Prandtl's research work in fluid mechanics, regardless of the fact that he made fundamental contributions to the other branches of mechanics (elasticity theory, plasticity).

2. SOME HIGHLIGHTS OF PRANDTL'S SCIENTIFIC WORK

2.1. *Boundary layer theory*

At the beginning of this century the view prevailed that it was hopeless to solve the Navier–Stokes equations of a viscous fluid, especially, as far as the problem of external flow about a body was concerned. For this reason Prandtl searched for approximate solutions for low-viscosity fluids, such as air and water, which are important in engineering applications. He succeeded to reduce the mathematical difficulties of the Navier–Stokes equations to such an extent that the simplified system of equations could be solved with the aid of the then existing methods. From a physical point of view, the simplification consists in the following:

Prandtl divided the complete flow field into two regions: the thin boundary layer which develops very close to the solid wall, in which the frictional forces are as important as the inertia forces; and the external region, in which the flow is practically frictionless.

What has just been said constitutes the foundation of Prandtl's boundary layer theory of 1904. The most important result of this theory is the calculation of the point of separation of the flow from the wall and the skin friction.

As an example of this theory we give a result for the laminar boundary layer on a flat plate. In this case there occurs no separation, because the pressure gradient is zero along the plate. The coefficient of the local skin friction is obtained as

$$\frac{\tau_0}{\rho U_\infty^2/2} = c_f' = \frac{0{\cdot}664}{\sqrt{(Re_x)}} \quad \text{(laminar, Blasius)} \qquad (1)$$

with ρ as density, U_∞ as velocity of the external flow, τ_0 as shearing stress at the wall, and $Re_x = U_\infty x/\nu$ as the local Reynolds number with x as the distance from the leading edge of the plate.

In Fig. 1 the coefficient of local skin friction of equation (1) is compared with experiments both for laminar and turbulent flow. The agreement between theory and experiment is excellent.

The concept of the boundary layer has also been very successfully applied to problems of heat transfer between a heated body in an external flow. In this case, in addition to the velocity boundary layer, there exists a temperature boundary layer. The calculation of the temperature boundary layer yields the heat transfer from the heated body to the external flow. An example of this kind is presented in Fig. 2 for a hot vertical flat plate in natural convection in air (Prandtl number $Pr = 0{\cdot}73$).

The upper diagram gives the temperature distribution in the boundary layer and the lower diagram the velocity distribution due to convection. Here, besides the Prandtl number, the Grashof number appears as parameter. It is given by the ratio of

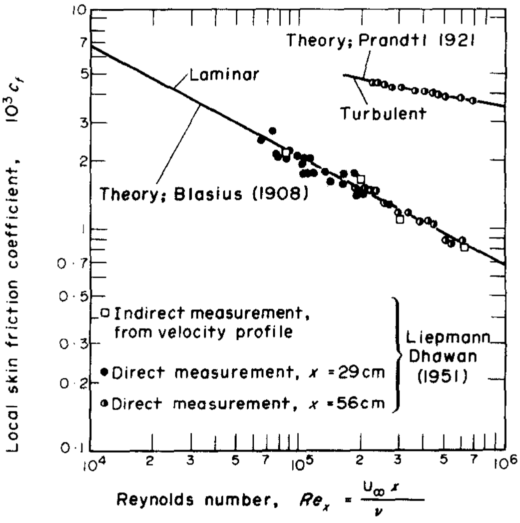

FIG. 1. Boundary layer of flat plate; local coefficient of skin friction (laminar, equation (1), Blasius, 1907).

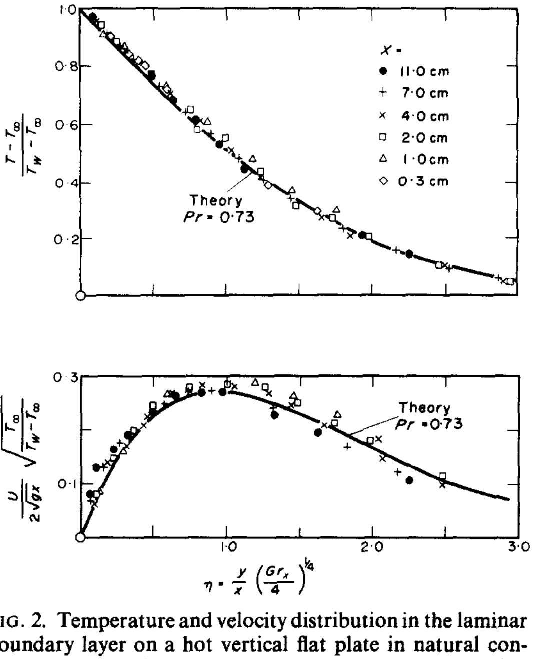

FIG. 2. Temperature and velocity distribution in the laminar boundary layer on a hot vertical flat plate in natural convection. Theory after E. Pohlhausen, 1921. Experiments after E. Schmidt and W. Beckmann, 1930.

buoyancy forces to inertial forces. This figure demonstrates that also for the temperature boundary layer there is very good agreement between theory and experiment.

2.2. *Turbulent flow*

The concept of the boundary layer which was applied by Prandtl and his collaborators at first to laminar flow proved to be very successful also for turbulent flow. A milestone in this respect was the resistance law for spheres and cylinders of circular cross-section. In the diagram of the drag coefficient against Reynolds number there is a sudden decrease of the drag coefficient with increasing Reynolds number at a critical Reynolds number. With his famous experiment using a tripping wire on a sphere Prandtl was able to demonstrate that the steep decrease of the drag coefficient at the critical Reynolds number is a consequence of the transition from laminar to turbulent flow in the boundary layer. Since the turbulent boundary layer is less sensitive to separation than the laminar boundary layer, the transition from laminar to turbulent flow shifts the separation point on the surface backwards and thus gives a smaller wake and a smaller drag.

For fully developed turbulent flow Prandtl introduced the concept of the mixing length, and thus was able to establish a semiempirical theory starting from Reynolds' apparent stresses due to the turbulent velocity fluctuations.

This mixing length concept, which was introduced by Prandtl as early as 1925, proved very useful both to flow problems in pipes and also for free turbulence as for instance in jets and wakes.

In Fig. 1 the skin friction for the turbulent boundary layer on the flat plate as obtained by theory and experiment is also given in complete agreement.

2.3. *Stability theory*

In addition to his numerous theoretical and experimental studies on the problems of developed turbulence, Prandtl occupied himself uninterruptedly in the twenties with the problem of the origin of turbulence. This problem had earlier, that is in the eighties of the preceding century, troubled Reynolds, after he had obtained his fundamental experimental results. The hypothesis that the appearance of turbulence is the result of an instability developed in the laminar motion can be traced to him (Reynolds' hypothesis). Nevertheless, almost half a century was needed to demonstrate the truth of this hypothesis, and a further twenty years elapsed before an experimental verification for this theory was produced. The mathematicians active in this field in the first decade of this century derived the Orr–Sommerfeld equation in the frame-work of a linear stability theory. They tried to demonstrate the existence of unstable laminar flow by seeking solutions of this equation with amplified superimposed disturbances. But the efforts of the mathematicians were unsuccessful for a long time. Prandtl and his collaborators started to work on this problem about 1920. However, a theory developed by Tietjens proved to be inadequate for the calculation of a critical Reynolds number for the boundary layer on a flat plate.

It was left to Prandtl's associate W. Tollmien to discover the decisive breakthrough that led to the solution of this problem in 1929. Tollmien was able, for the first time, to calculate theoretically a critical Reynolds number for the boundary layer on a flat plate at zero incidence in good agreement with experiment. In the years to follow, H. Schlichting extended this theory and proved that the critical Reynolds number depends strongly, among others, on the pressure gradient along the wall.

It is strange to report that these theoretical results for a long time were not completely accepted by the specialists working outside Germany. Fourteen years were to elapse until the careful measurements performed by H. L. Dryden and his colleagues fully confirmed this theory in the year 1943.

2.4. *Experimental technique* (*wind tunnels*)

A short time after his arrival in Göttingen, namely towards the end of the year 1906, Prandtl's thoughts turned in a completely new direction. This development was to prove as important for his scientific attainments as his efforts on behalf of flows with separation and turbulence. This was the dawn of the new era of aeronautics. Otto Lilienthal executed in 1896 the first gliding flight. In the United States, the brothers Wright achieved the first powered flight in the year 1903.

At the beginning of the twentieth century, German engineers were working on the development of the airship. This task was supported by the Motor Luftschiff Studiengesellschaft (Society for the Study of Airships). Prandtl participated in this development—again with Felix Klein's encouragement—in that he designed an installation for the testing of models in a wind tunnel. This wind tunnel came into operation in 1908. It operated in competition with another type of wind tunnel constructed simultaneously by G. Eiffel in Paris and became so successful that other wind tunnels were modelled after it in many countries.

The essential characteristic of the first wind tunnel of 1908 is that the air circulates in a closed duct. The important parameters were:

$$\left.\begin{array}{ll}\text{Working section:} & 2\times 2 = 4\,\text{m}^2\\ \text{Maximum speed:} & 10\,\text{m/s} = 36\,\text{km/h}\\ \text{Power installed:} & 30\,\text{hp.}\end{array}\right\}\begin{array}{c}\text{First tunnel}\\ 1908\end{array}$$

During World War I Prandtl built his second wind tunnel, together with his collaborator A. Betz, which came into operation in 1917. This tunnel was equipped with a nozzle of large contraction ratio, and its maximum speed was considerably higher than that of the first tunnel. The important characteristics of the second Göttingen tunnel were:

$$\left.\begin{array}{ll}\text{Working section:} & 4\,\text{m}^2 (D = 2{\cdot}25\,\text{m})\\ \text{Maximum speed:} & 50\,\text{m/s} = 180\,\text{km/h}\\ \text{Power installed:} & 400\,\text{hp.}\end{array}\right\}\begin{array}{c}\text{Second tunnel}\\ 1917\end{array}$$

This type of wind-tunnel (the so-called Goettingen type), conceived by Prandtl, soon became one of the most important experimental aids for the performance of all manner of experiments on bodies in flow. This tunnel exerted in all countries a large influence on the development, on the science of aerodynamics, and the art of aeronautical engineering.

2.5. *Wing theory*

The wind tunnels just described were not only for investigations on airships, but also for airfoils. As a result of his work in the field of the aerodynamics of wings undertaken during the first decade of this century, Prandtl found himself in competition with F. W.

Lanchester of England. Both endeavored, at first independently of each other, to determine the effect of finite span of a wing on its aerodynamic properties. In close relation with this problem, they attempted to explain the so-called induced drag around the edges of the wing. Carl Runge, the other important mathematician of Göttingen, deserves to be given the credit for having brought together in Göttingen the two aerodynamicists—Prandtl and Lanchester—for a scientific discussion. It was also on Runge's initiative that Lanchester's book concerning the aerodynamics of flight was translated from the English into the German language.

The continuous efforts expanded by Prandtl to understand the observed characteristics of flows, to describe them, as far as possible, theoretically and numerically, and to explore them by the scientific method, characterized all publications which appeared during these pioneering times.

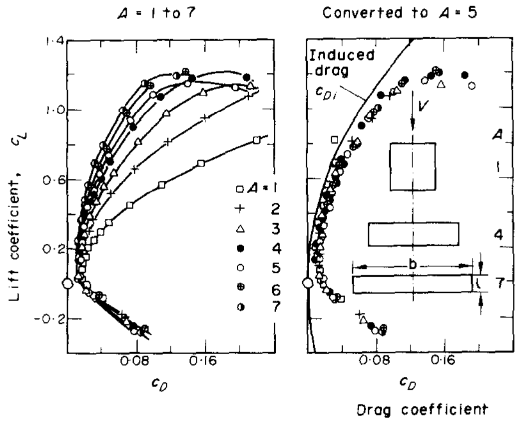

FIG. 3. Wing of finite span; polar curves for different aspect ratios (Prandtl–Betz, 1921). $A = 6/1$ = aspect ratio.

In the years 1918 and 1919 Prandtl submitted two papers to the Göttingen Academy of Science on his discoveries concerning wing theory. This publication contained Prandtl's theory of the lifting line for a wing of finite span, the theory of the multiplane, the theory of the wing of minimum induced drag, as well as many other contributions. This is an achievement of the same rank as his boundary layer theory of 1904. I shall report here one result, which is most important from the practical point of view. In Fig. 3 Prandtl's classical result on the lift and drag of an airfoil as a function of the aspect ratio A is given. For the drag coefficient, c_D, in terms of the lift coefficient, c_L, he could prove that

$$c_{D2} = c_{D1} + \frac{c_L^2}{\pi}\left(\frac{1}{A_2} - \frac{1}{A_1}\right), \tag{2}$$

subject to the condition that A is large. In this figure the measured curves of lift coefficient against drag coefficient are given for five different aspect ratios A as five separate curves. However, when they are reduced with the aid of equation (2) to an aspect ratio $A = 5$, they trace a single curve. Thus, experiment brilliantly confirmed Prandtl's wing theory.

Prandtl has also contributed numerous papers to compressible flow (gas dynamics) and meteorological problems; but space does not allow to deal with these here.

His own publications, including his books and contributions to hand-books, sum up to the considerable number of 168; the number of doctorates reached 83.

3. PRANDTL AS A UNIVERSITY PROFESSOR

Prandtl's career as an active university professor extended over a period of more than 45 years, because he was appointed at the young age of 26 years to an ordinary professorship of mechanics at the Technical University of Hanover in 1901. At Felix Klein's behest, Prandtl had accepted in 1904 the invitation extended to him by the University of Göttingen to occupy the newly created chair of applied mechanics. In 1947 he retired from the professorship at Göttingen at the age of 72. During the whole of this long period of time, Prandtl applied himself to his lectures with great devotion. He remained faithful to Göttingen to the end of his life. Prandtl repeatedly stressed the fact that the link between the pure and the applied science was for him important as well as indispensable and that the cultivation of this spirit was a unique feature of life at Göttingen University. A large number of students was introduced by him to the science of mechanics, and, in particular, of fluid mechanics. Many of them were induced to continue as doctoral candidates and to work under Prandtl in one of the institutes directed by him.

I think, that this account clearly proves that the science of mechanics, particularly the science of fluid mechanics, progressed enormously through Ludwig Prandtl; he placed, so to say, this science in a new epoch.

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